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Single-Parameter Terrain Classification for Terrain Following

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To avoid excessive simulation of terrain following flights, it is desirable to have a method to classify terrain with regard to its suitability for terrain following. Analytical expressions for the probability of crashing (P_c) for a terrain-following missile indicate that the variances σ_e^2 and $\sigma^2_{e'}$ of missile altitude error and error rate, respectively, have considerable effect on P_c. In general, the larger they are, the larger is P_c. Linearized studies of the missile control system interacting with stationary terrain statistics indicate that both σ_e and $\sigma_{e'}$ vary with σ_T, the standard deviation of the terrain, and with some function of terrain bandwidth. This suggests that even for the nonlinear, nonstationary case, the ratio $R=\sigma_T/l_T$ where l_T is the terrain correlation length has a large effect on missile terrain-following performance. The parameter R is called the roughness ratio. Actual digitized terrain was processed to obtain moving estimates of R. This provides a single-parameter preliminary classification method for a variety of terrains. Simulated flights over strips of terrain confirmed that in general the missile had most difficulty where R was largest.

Nomenclature

$c_{xx}(k,n), c^l_{xx}(k,n)$	= moving estimates of autocovariance, correlation function, ending at point n
$C^l(z_1,z_2)$	= two-dimensional z-transform of $c^l_{xx}(k,n)$
$E(t), e(t)$	= missile altitude error random process, realization, ft
$E'(t), e'(t)$	= missile altitude error rate random process, realization, ft/s
f_1, f_c, f_s	= fundamental frequency, cut off frequency, sampling frequency, cycles/s
f_T	= terrain bandwidth, cycles/s
g_1, g_2	= functions of θ according to Table 1
$h(t), h_{CL}(t), h_0$	= missile altitude, clearance altitude, commanded altitude, ft
$H_T(t), h_T(t)$	= terrain altitude random process, realization, ft
k	= lag
m	= number of elevation points in moving window
n	= point at which moving estimate ends
N	= number of sample points in a given digitized terrain strip
N_M, N_T	= order of missile altitude control system, terrain
N_S	= number of downrange strips in digitized terrain map
P_c	= probability of crashing
$r_{xx}(k,n)$	= moving estimate of normalized autocovariance
R	= roughness ratio of terrain $=\sigma_T/l_T$
$\hat{R}(n)$	= moving estimate of $R=\hat{\sigma}_T(n)/\hat{l}_T(n)$
T	= flight time, s
T_s	= cell size or sampling period for digitized terrain
$\bar{x}(n)$	= moving estimate of sample mean
$Y(z), V(z), H(z)$	= two-sided z-transforms of sequences $y(n), v(n), h(n)$
V	= missile speed, ft/s
$l_T, \hat{l}_T(n)$	= correlation length of terrain, moving estimate
ρ	= correlation coefficient
$\sigma_e, \sigma_{e'}$	= standard deviation of altitude error, error rate, ft
$\sigma_T, \hat{\sigma}_T, \hat{\sigma}_T(n)$	= standard deviation of terrain, estimate, moving estimate, ft
θ	= missile/terrain bandwidth ratio $=\omega_M/\omega_T$
ω	= frequency variable, rad/s
ω_c	= cutoff frequency, rad/s
ω_s	= sampling frequency, rad/s
ω_M, ω_T	= bandwidth of missile altitude control system, terrain, rad/s

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Introduction

THE nature of the terrain over which a missile is flying in a terrain-following mode, will have a decisive effect on the success or failure of its mission. Detailed simulation of the missile flying over a particular terrain will indicate the clearance command schedule required for a low probability of crashing.[1] However, because of the expense involved, it is clearly impractical to simulate flights over all terrains of interest. Instead, some method is needed for classifying terrain. The objectives of such a method are twofold: 1) to determine its suitability for terrain following, and 2) to generate clearance command schedules without excessive simulation of actual missile flights.

A classification method which has been used from the earliest terrain-following studies[2] involves dividing large regions up into subregions according to their $\hat{\sigma}_T$ values. The parameter $\hat{\sigma}_T$ is an estimate of the terrain standard deviation σ_T. Typically, it was computed over an area (a matrix of elevation points) or over a strip (a row of elevation points). In either event, it is, in this form, a poor classification method for terrain following. Simulation studies have shown that, in some cases, it was easier to fly over one strip than over another although the $\hat{\sigma}_T$ value for the first was greater than that for the second strip. The averaging process involved in estimating the single $\hat{\sigma}_T$ value may disguise terrain features which would be troublesome to the missile. In addition, the estimate of $\hat{\sigma}_T$, so obtained, does not take into account the frequency content of the terrain.

In Ref. 3, McGlynn and Sovine propose a scheme which fits a second-order autoregressive model[4,5] to all terrain. The two parameters of this model are then used to classify the terrain. However, studies at APL and elsewhere indicate that the terrain order may vary, in general, from first to sixth with a corresponding number of parameters required for classification.

Classification by means of a single parameter which can be computed from the terrain itself is the goal here. The parameter should be relatively easy to compute and should provide a means for a broad preliminary classification of a variety of terrain types. Then, simulated terrain-following flights may be made over selected areas deemed suitable by the preliminary classification. A candidate parameter is proposed in this paper. It was checked with simulated flights, the results of which are very promising.

Derivation of the Classification Parameter

The terrain-following situation is depicted in Fig. 1. The missile is assumed to be controlled to a certain altitude h_0 above the terrain, flying at speed V which is approximately constant. Since, in advance of flight, it is not known exactly which particular strip the missile will fly over, it is reasonable, for analysis purposes, to regard contiguous strips $h_T(t)$ as realizations of a random process $H_T(t)$. It follows that the altitude error $e(t)$ can also be regarded as a realization of a random process $E(t)$.

In Ref. 1, it was shown that under the assumptions that $E(t)$ is stationary and normally distributed, the probability of crashing in T seconds is

$$P_c(h_0,T) = 1 - \exp\left[-\frac{1}{2\pi}\frac{\sigma_{e'}}{\sigma_e}\exp\left(\frac{-h_0^2}{2\sigma_e^2}\right)T\right] \tag{1}$$

where σ_e and $\sigma_{e'}$ are the standard deviations of error and error rate, respectively. It is evident from Eq. (1) that the dominant parameter affecting P_c is σ_e, for a given h_0. The greater σ_e, the greater is P_c. It has been confirmed by simulation[6] that this is true also if $E(t)$ is stationary but not normally distributed. Furthermore, P_c increases with $\sigma_{e'}$ but the effect is not so pronounced.

For the purposes of terrain classification, it is desirable to determine what terrain parameters affect $\sigma_{e'}$ and $\sigma_{e'}$. Perhaps the best guide is provided by the results of a linearized study[1] giving missile response variances as a function of

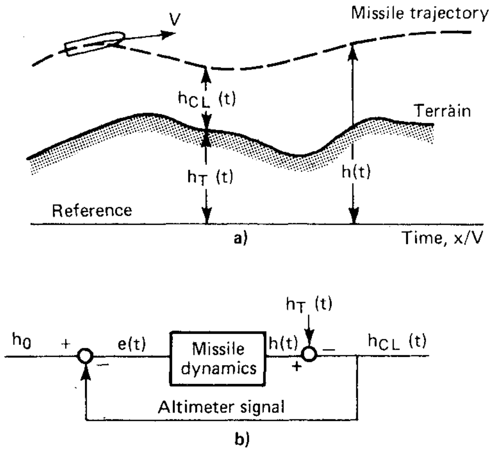

Fig. 1 Terrain following.

missile/terrain bandwidth ratio θ. These are summarized for some simple cases in Table 1. The first two rows of the table give the desired information. From the first row

$$\sigma_e = \sigma_T g_1(\theta) \tag{2}$$

and from the second row

$$\sigma_{e'} = \sigma_T \omega_T g_2(\theta) \tag{3}$$

where

$$\theta = \omega_M/\omega_T$$

and ω_M and ω_T are the bandwidths of the missile altitude control loop and the terrain, respectively. Thus, both σ_e and $\sigma_{e'}$ are directly proportional to σ_T. They also depend on functions g_1 and g_2 of θ as defined in Table 1. For example, from the first row of Table 1, with a second order altitude control system and a second order terrain,

$$\sigma_e^2/\sigma_T^2 = (1+3\theta+2\theta^2)/(1+2\theta+2\theta^2+2\theta^3+\theta^4) \tag{4}$$

It is often true that the bandwidth ω_M of the altitude control loop is considerably greater than the terrain bandwidth ω_T. In this case, the higher powers of θ dominate in Eq.

Table 1 Missile response variances as functions of missile/terrain bandwidth ratio

Order of altitude control system N_M	1			2		3
Terrain Order N_T	1	2	3	1	2	1
$\frac{\sigma_e^2}{\sigma_T^2}$	$\frac{1}{1+\theta}$	$\frac{1}{1+(2)^{1/2}\theta+\theta^2}$	$\frac{2+\theta}{2[1+2\theta+2\theta^2+\theta^3]}$	$\frac{1+[3/(2)^{1/2}]\theta}{1+(2)^{1/2}\theta+\theta^{2-}}$	$\frac{1+3\theta+2\theta^2}{1+2\theta+2\theta^2+2\theta^3+\theta^4}$	$\frac{3+8\theta+10\theta^2}{3[1+2\theta+2\theta^2+\theta^3]}$
$\frac{\sigma_{e'}^2}{\omega_T^2\sigma_T^2}$	...	$\frac{1+(2)^{1/2}\theta}{1+(2)^{1/2}\theta+\theta^2}$	$\frac{1+2\theta}{2[1+2\theta+2\theta^2+\theta^3]}$	...	$\frac{1+2\theta+4\theta^2+3\theta^3}{1+2\theta+2\theta^2+2\theta^3+\theta^4}$	...
$\frac{\sigma_{e'}^2}{\omega_T^2\sigma_e^2}$	...	$1+(2)^{1/2}\theta$	$\frac{1+2\theta}{2+\theta}$	...	$\frac{1+2\theta+4\theta^2+3\theta^3}{1+3\theta+2\theta^2}$	...
$\frac{\sigma_h^2}{\sigma_r^2}$	$\frac{\theta}{1+\theta}$	$\frac{\theta[(2)^4+0]}{1+(2)^1\theta+\theta^2}$	$\frac{\theta[3+4\theta+2\theta^2]}{2[1+2\theta+2\theta^2+\theta^3]}$	$\frac{\theta\{[1/(2)^{1/2}]+\theta\}}{1+(2)^{1/2}\theta+\theta^2}$	$\frac{\theta[1+2\theta+2\theta^2+\theta^3]}{1+2\theta+2\theta^2+2\theta^3+\theta^4}$	$\frac{\theta[2+4\theta+3\theta^2]}{3[1+2\theta+2\theta^2+\theta^3]}$

[a] $\theta=\omega_M/\omega_T=$ missile bandwidth/terrain bandwidth.

(4) and

$$\sigma_e^2/\sigma_T^2 \approx 2/\theta^2 \qquad (5)$$

or

$$\sigma_e \approx k_1 \sigma_T/\theta \qquad (6)$$

Missile bandwidth does not vary much over the length of a map to be processed and so Eq. (6) becomes

$$\sigma_e \approx k_2 \sigma_T \omega_T \qquad (7)$$

To estimate ω_T, one can perform a spectral analysis of a particular terrain strip and define the bandwidth as, for example, the band between the half-power points of the spectral estimate. It requires less computation to work instead with the correlation length l_T of the terrain which bears an inverse relationship to ω_T, that is, the wider ω_T, the shorter is l_T. Arbitrarily define l_T as the lag in time or distance before the autocovariance function of the terrain drops to 0.368 (e^{-1}) at its value at zero lag, whatever the order of the terrain. Thus, Eq. (7) may be written

$$\sigma_e \approx k_3 \sigma_T/l_T \qquad (8)$$

The ratio

$$R = \sigma_T/l_T \qquad (9)$$

will be called the "roughness ratio" of the terrain.

Equation (8) implies that the larger the roughness ratio, the larger is σ_e and the more likely the missile will crash. This was derived for a specific example. Examination of the other entries in the first two rows of Table 1, reveals that σ_e depends either on σ_T/l_T or $\sigma_T/l_T^{1/2}$ and $\sigma_{e'}$ depends either on $\sigma_T/l_T^{3/2}$ or σ_T/l_T^2 for large θ. Higher order examples behave similarly. If the ratio σ_T/l_T is largest at a specific point in a strip as compared with other points in that strip, then the ratio $\sigma_T/l_T^{1/2}$, for example, is also largest there. There seems to be little point, then, in refining the classification by taking a ratio other than σ_T/l_T into account. Possibly, in some special cases, such a refinement might be deemed desirable to account for different orders of terrain in a given map. In addition, it should be noted that in cases where missile bandwidth is not considerably greater than terrain bandwidth it is necessary to use Eqs. (2) and (3).

The result in Eq. (8) based on linearized analysis, assuming stationary terrain statistics, suggests a means whereby a given region of terrain may be classified for terrain following. This is discussed in the next section.

Map Processing for Terrain Classification

Suppose a map is available consisting of a rectangular matrix of elevation points which defines a certain region of terrain. The distance between points is the cell size or sampling period T_s. It is desired to classify the map with regard to its difficulty for terrain following.

It is evident that the classification must be directional, that is, an approximate flight direction must be assumed for the missile over the map. If this is not known then it is necessary to classify the map in at least two directions at right angles. The results of the previous section suggest that even though the statistics of a given map often contain low frequency nonstationary trends and the missile altitude control loop in nonlinear, the parameter $R = \sigma_T/l_T$ should have considerable effect on the terrain-following characteristics of the map.

Assume the flight direction is in the long direction of the map. If the map statistics were stationary, estimates of σ_T and l_T could be obtained by calculating these quantities for each downrange map strip and then taking an average over the ensemble of strips. There are two drawbacks to this approach. First, as mentioned above, the map is generally nonstationary especially if low frequency trends are not filtered out. Second, even if it were stationary, the ensemble averages would yield parameter estimates which would relate to the average behavior of the missile if it were to fly over many strips of the map whereas, interest is in its performance over a particular strip.

Hence, the map is processed strip by strip and to allow for the nonstationary aspects of the terrain, moving estimates of σ_T and l_T are taken over a limited number m of points along the strip. In this manner, it can be determined how the roughness ratio varies along the strip. The reasoning is that, while the statistics may be nonstationary over the full strip, they will be more nearly stationary over smaller subintervals of the strips because as mentioned above, the nonstationarities are generally at low frequencies.

Moving estimates of σ_T and l_T are obtained as follows. Let $x(1),\ldots,x(N)$ be a particular map strip where N is the number of points in the strip. The number of strips is N_s. The autocovariance function, estimated for a moving "window" of length m points, $m<N$, is

$$c_{xx}(k,n) = \frac{1}{m} \sum_{i=n_1}^{n-k} \left[x(i) - \bar{x}(n)\right]\left[x(i+k) - \bar{x}(n)\right] \qquad (10)$$

where

$$\bar{x}(n) = \frac{1}{m} \sum_{i=n_1}^{n} x(i) \qquad (11)$$

and $n = n_1 + m - 1 \leq N$. The variance estimate for the subinterval is

$$\hat{\sigma}_T^2(n) = c_{xx}(0,n) \qquad (12)$$

and the normalized autocovariance estimate is

$$r_{xx}(k,n) = \frac{c_{xx}(k,n)}{c_{xx}(0,n)} \qquad (13)$$

Define the correlation length estimate $\hat{l}_T(n)$ for the subinterval as the delay before the normalized autocovariance estimate falls to 0.368 of its zero delay value which is unity. Then the roughness ratio estimate for the (n_1,n) subinterval is

$$\hat{R}(n) = \frac{\hat{\sigma}_T(n)}{\hat{l}_T(n)} \qquad (14)$$

The argument n is used since it denotes the point in the strip at which the computation is completed. Note that the estimates $\hat{\sigma}_T(n)$, $\hat{l}_T(n)$ and $\hat{R}(n)$ are for the m points up to and including n. Thus, m points along a strip are processed before the first estimates can be obtained.

A complete processing of the map would involve starting with the first strip, computing $\hat{\sigma}_T$, $\hat{l}_T$, and $\hat{R}$ for $n_1 = 1$, $n = m$, then for $n_1 = 2$, $n = m+1$, up to $n_1 = N-m+1$, $n = N$ and doing the same for the other strips until all N_s strips have been processed. Then the points where the highest values of $\hat{R}$ occur can be noted and a decision made either to program a higher clearance command near those points or to avoid them altogether.

To reduce the amount of computation, it may suffice in many cases, depending upon the cell size, to take moving estimates perhaps every 10 cells rather than every cell along a strip and also to process, say, every 10th strip rather than every strip across the map.

Determining the Window Length

The operation of taking a moving average of a sequence $\{v(i)\}$, $i = 1,2,\ldots$ may be analyzed by rewriting the expression

$$y(n) = \frac{1}{m} \sum_{i=n_1}^{n} v(i) \qquad (15)$$

where $n_1 = n - m + 1$, as

$$y(n) = \frac{1}{m} \sum_{i=0}^{m-1} v(n-i) \tag{16}$$

Taking the z-transform of Eq. (16) gives

$$Y(z) = H(z)\, V(z) \tag{17}$$

where

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \tag{18}$$

$$V(z) = \sum_{n=\infty}^{\infty} v(n) z^{-n} \tag{19}$$

and

$$H(z) = \frac{1}{m} \frac{1 - z^{-m}}{1 - z^{-1}} \tag{20}$$

Thus, the operation of taking the moving average of a sequence is equivalent to passing the sequence through a digital filter with system function $H(z)$ given by Eq. (20). Setting $z = \exp(j\omega T_s)$ in Eq. (20), gives the frequency response as

$$H(e^{j\omega T_s}) = \frac{1}{m} \exp[-j(m-1)\omega T_s/2] \frac{\sin(m\omega T_s/2)}{\sin(\omega T_s/2)} \tag{21}$$

where $|\omega| \leq \omega_s/2$ and $\omega_s = 2\pi/T_s$ is the radian sampling frequency. This is in the form of a low pass filter with cutoff frequency $\omega_c = \omega_s/m$.

Because of the upper limit $n-k$ and the time-varying mean, it is difficult to determine the frequency response associated with the moving estimate of the autocovariance function defined by Eq. (10). However, if the mean is not subtracted (i.e., the autocorrelation function is considered) and it is assumed that data exist for $i > n$, as it will in most cases, a moving estimate of the autocorrelation function can be defined as

$$c_{xx}^1(k,n) = \frac{1}{m} \sum_{i=n_1}^{n} x(i)x(i+k) \tag{22}$$

With change of variable, this becomes

$$c_{xx}^1(k,n) = \frac{1}{m} \sum_{i=0}^{m-1} x(n-i)x(n-i+k) \tag{23}$$

This is a two-dimensional sequence in n and k. Taking the two-dimensional z-transform gives

$$C^1(z_1,z_2) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c^1(n,k) z_1^{-n} z_2^{-k}$$

$$= H(z_1) X_{1,2}(z_1, -z_2) X_1(z_2) \tag{24}$$

where

$$X_{1,2}(z_1, -z_2) = \sum_{r=-\infty}^{\infty} x(r) z_1^{-r} z_2^{r} \tag{25}$$

$$X_1(z_2) = \sum_{r=-\infty}^{\infty} x(r) z_2^{-r} \tag{26}$$

and $H(z_1)$ is as defined in Eq. (20) with z replaced by z_1.

If z_1 is replaced by $\exp(j\omega_1 T_s)$ and z_2 by $\exp(j\omega_2 T_s)$, one obtains the two-dimensional frequency response version of Eq. (22). Again it will be noted that the spectral window $H(e^{j\omega T_s})$ as defined in Eq. (21) appears explicitly to indicate a low pass filtering operation.

The foregoing provides a guide as to a suitable window length m for a given strip of terrain of length NT_s. It is assumed that N and T_s are chosen so as to provide a good indication of the frequency content of the terrain. The frequency resolution, or fundamental frequency, is $f_1 = 1/NT_s$. The cut-off frequency for the low pass filtering defined by the moving average operation is $f_c = f_s/m$ where $f_s = 1/T_s$. If $m = N$ so that, for example, the σ_T estimate is taken over the whole strip as in previous studies, then $f_c = f_s/N = f_1$. This is clearly unsuitable as a descriptor of terrain roughness since all the harmonics are attenuated or filtered out. Instead, m should be chosen less than N and have a value such that the power at the important frequencies of the terrain spectrum is not attenuated. A rough guide is to select m so that $f_c > f_T$ where $f_T = \omega_T/2\pi$ is the terrain bandwidth. Since $m = f_s/f_c$, it follows that m should be selected so that

$$m < f_s/f_T \tag{27}$$

If there are no other constraints, it is desirable to choose the window length close to the bound set by inequality (27), to reduce computation. In general, however, where there are many strips of a particular map to be processed, each of which may have a different estimate of bandwidth f_T, it is better to choose a single m for the whole map on the basis of the strip with largest bandwidth estimate. This provides uniformity for comparison of the roughness ratio values for the entire map.

Some Test Results

The above method was used successfully to classify several terrain areas for terrain following. One of the maps will be covered here. Moving estimates of σ_T, l_T, and R were made every 10 cells along every 10th strip. Thus, there were many voids in the complete classification of the map, but the data obtained provided much insight into the suitability of the processed strips for terrain following.

On Fig. 2 are shown the results for a particular strip ($J = 8$). The strip was divided into three sections. The first section had by far the largest values of roughness ratio $\hat{R}$ due to the short correlation lengths there. The second two sections had considerably higher $\hat{\sigma}_T$ values than the first but $\hat{R}$ was lower there because of the corresponding longer correlation lengths. Thus, in view of the foregoing analysis the missile would be expected to have more difficulty in the first section.

This was confirmed by simulating a flight of the missile along the strip. Various nonlinearities were included in the altitude control loop. The clearance command was constant. Statistics of altitude error and error rate were collected for each section. The minimum clearance in each section and the point at which it occurred were noted. The latter are marked on Fig. 2. By far the smallest of the 3 minima occurred in section 1. Furthermore, the statistics indicated that, using the methods of Ref. 1, the probability of crashing was considerably higher in section 1 than in the other two sections. Terrain-following performance for the latter sections was roughly equivalent, confirming the relative $\hat{R}$ values in those sections. Note that the values of the parameters $\hat{\sigma}_T$, $\hat{l}_T$, and $\hat{R}$ at a point are computed for the m cells (100 in this case) up to and including that point and so the effects may be felt by the missile anywhere in that span of terrain.

Similar results for another strip ($J = 9$) are shown in Fig. 3. Again the higher $\hat{\sigma}_T$ values occurred in sections 2 and 3 but the greatest values of $\hat{R}$ were in section 1. In fact, the peak $\hat{R}$ value which occurred at $n = 160$ was the highest value of the ratio noted in processing the map. Terrain-following flights

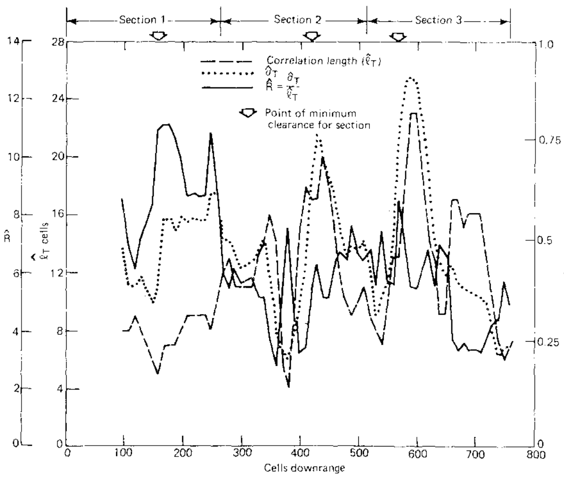

Fig. 2 Moving estimates of σ_T, l_T, and ratio ($J=8$, 100 cell estimates, every 10 cells).

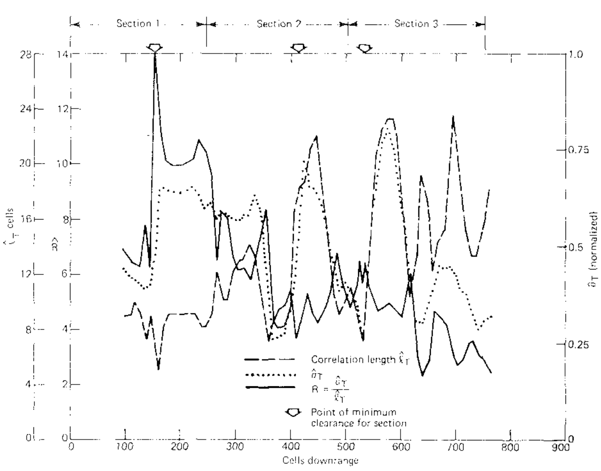

Fig. 3 Moving estimates of σ_T, l_T, and ratio ($J=9$, 100 cell estimates, every 10 cells).

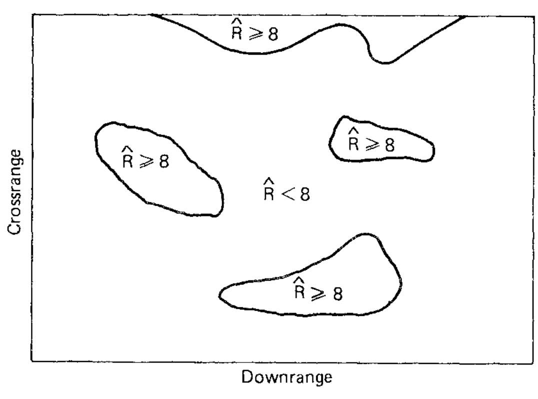

Fig. 4 Example of partial classification of map, $R \lessgtr 8$.

confirmed the difficult nature of the terrain near that point. The missile almost crashed.

An example of partial classification of the map in question is shown in Fig. 4. The map is divided into areas with the ranges of $\hat{R}$ values noted. In general, the higher the $\hat{R}$ values, the rougher the terrain and the greater the required clearance command for the missile.

As mentioned, Fig. 4 provides only partial classification of the terrain in question. A complete classification would entail processing every strip to obtain $\hat{R}$ at each point in the strip as described earlier. The problem in not making a complete

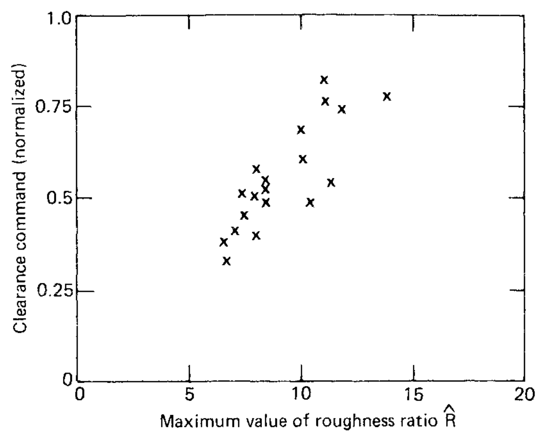

Fig. 5 Scatter diagram of clearance command vs roughness ratio for a fixed P_c.

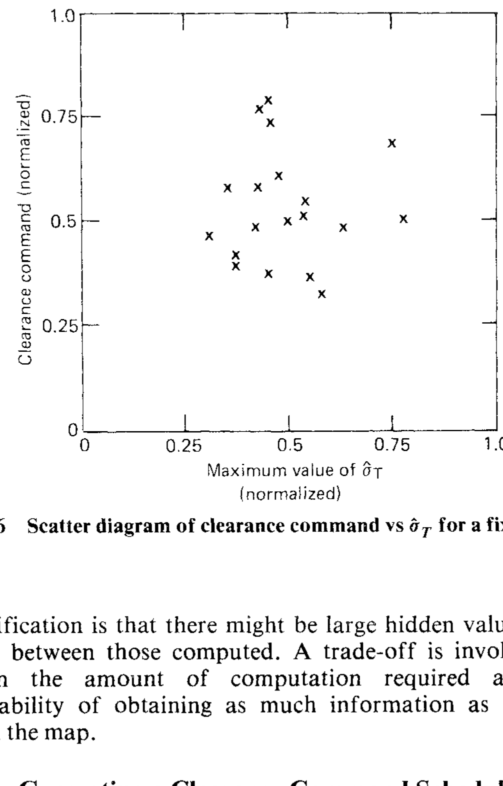

Fig. 6 Scatter diagram of clearance command vs $\hat{\sigma}_T$ for a fixed P_c.

classification is that there might be large hidden values of $\hat{R}$ lying between those computed. A trade-off is involved between the amount of computation required and the desirability of obtaining as much information as possible from the map.

Generating a Clearance Command Schedule

One of the primary objectives in classifying terrain for terrain following is to generate a schedule of clearance commands. Because of the wide variation in terrain characteristics, it is unlikely that any finite classification could cover all possible situations. However, using the method illustrated above, it is possible to obtain a broad classification which, together with a limited amount of flight simulation, enables one to ascertain in advance the clearance command to be programmed to achieve a desired probability of crashing (P_c).

As an illustration of how this may be achieved consider Fig. 5, a scatter plot of normalized clearance command h_0 vs maximum value of roughness ratio based on terrain-following flights simulated over different terrains. The clearance command is that required for a certain selected value of P_c. There is strong positive correlation between h_0 and $\hat{R}$ (correlation coefficient $\rho \approx 0.8$ to 0.9). A straight line through the origin with half the points above and half below would give a rough indication of the required h_0 for a given maximum value of $\hat{R}$. Of course, it would be oversimplifying matters to assume that h_0 is a linear function of the maximum $\hat{R}$ value. Among other factors, the terrain order would also be

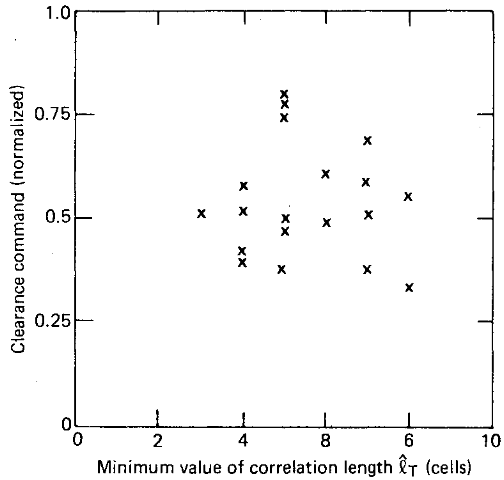

Fig. 7 Scatter diagram of clearance command vs correlation length for a fixed P_c.

taken in account in a more complete analysis. However, the correlation between h_0 and maximum $\hat{R}$ value indicates that the latter serves as a rough guide for selecting the required clearance command as well as providing a means for preliminary classification of a broad variety of terrains.

Figures 6 and 7 are based on the same set of simulated flights. Figure 6 is a scatter diagram of clearance command vs maximum $\hat{\sigma}_T$ value for a fixed P_c. There is surprisingly poor correlation between the two parameters ($\rho<0.5$). A similar plot of h_0 vs $\hat{l}_T$ is shown in Fig. 7. One might expect a negative correlation between the two, that is, a small $\hat{l}_T$ minimum requires a large h_0. However, this is not evident from the figure. These two figures taken in conjunction with Fig. 5 tend to confirm that the ratio $\hat{R}$ of $\hat{\sigma}_T$ and $\hat{l}_T$ rather than the value of either has the greatest effect on the clearance command.

Conclusions

The results indicate that the roughness ratio R is a readily-computable parameter which may be used to provide a preliminary screening and classification of a broad variety of terrain with regard to its suitability for terrain following. This would eliminate the necessity for terrain-following simulation in unsuitable areas.

Having determined by selective simulation a clearance command setting for a particular range of $\hat{R}$ values, it is possible to extrapolate this setting to terrain having the same range of $\hat{R}$, without further flight simulation.

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